Compositional Verification for Nature-Derived Material Design

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We introduce a categorical framework for translating biological stimulus–response hierarchies into verified fabrication specifications, together with a computational implementation that makes the construction executable. The framework is organized as a four-category pipeline

$$\mathbf{Nat} \xrightarrow{\mathcal{F}} \mathbf{Art} \xleftarrow{\pi} \mathbf{Spec} \xrightarrow{\mathbf{E}} \mathbf{Comp},$$

where **Nat** and **Art** are subcategories of a shared category **Dyn** of stimulus–response systems, **Spec** is a category of fabrication specifications organized over behavioral targets in **Art**, and **Comp** is a category of executable machine programs. The central result is that once local interface conditions are established at each scale, system-level validity follows by composition without re-validation at the assembled level.

Categorical setup. We define a category **Dyn** of stimulus–response systems [1]. An object is a triple $S = (X, E, f)$ with X a smooth finite-dimensional state manifold, E an environment manifold, and $f\colon X \times E \to TX$ a controlled vector field. A morphism $(\alpha, \alpha_E)\colon S \to T$ is a pair of smooth maps satisfying the simulation condition

$$d\alpha_x\big(f(x,e)\big) \;=\; g\big(\alpha(x),\, \alpha_E(e)\big) \qquad \forall\, (x,e) \in X \times E.$$

Biological and engineered systems form subcategories $\mathbf{Nat}, \mathbf{Art} \subset \mathbf{Dyn}$, distinguished by the physical origin of their dynamics. The implementation functor $\mathcal{F}\colon \mathbf{Nat} \to \mathbf{Art}$ must assign to each morphism in **Nat** a morphism in **Art** satisfying the simulation condition in **Art**, which is the engineering constraint that every interface between scales in the biological hierarchy be faithfully reproduced in the engineered system.

Fabrication semantics. Objects of **Spec** are fabrication programs $\Sigma = (D, P)$, where $D \subset \mathbb{R}^3$ is a design domain and P is a finite ordered collection of deposition primitives carrying annotations for material identity, layer position, raster orientation, and process parameters. Annotations partition into *constitutive* ones, which determine the **Art** evolution law, and *process* ones, which range freely within material-dependent admissible intervals. The semantic projection $\pi\colon \mathbf{Spec} \to \mathbf{Art}$ extracts the constitutive annotations and returns the corresponding **Art** object, sending every **Spec**-morphism to an identity. The subcategory $\pi^{-1}(A) \subset \mathbf{Spec}$ over a target $A \in \mathbf{Art}$ is the subcategory of programs realizing the same behavioral target. Morphisms within it are either process refinements, which adjust execution parameters without altering the constitutive model, or constitutive equivalences, which substitute material or geometry while leaving the extracted parameters unchanged. The execution functor $\mathbf{E}\colon \mathbf{Spec} \to \mathbf{Comp}$ compiles a verified specification into executable machine code and is forgetful in the sense that the physics encoded in π has no representation in **Comp**.

Verification stability and compositionality. The main structural result is *verification stability.* If $\Sigma \in \pi^{-1}(A)$ and $\varphi\colon \Sigma \to \Sigma'$ is any morphism in **Spec**, then $\Sigma' \in \pi^{-1}(A)$. This separates the verification problem of identifying the correct subcategory from the manufacturing problem of navigating within it. Because the pipeline is functorial throughout, local verification at each interface implies global verification and correctness of the assembled system follows by composition without further case analysis.

Instantiation and generative design. We instantiate the framework on the hygromorphic pinecone [2], a canonical multiscale system in which stimulus-driven shape change arises from a lower-scale constitutive asymmetry that propagates compositionally through lamina, tissue, element, and organ.

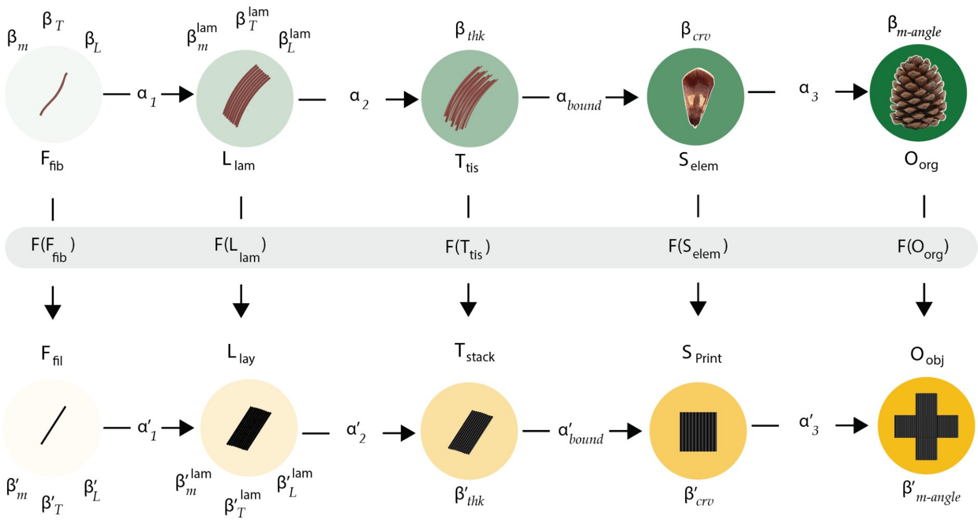

Figure 1: Diagrammatic translation under $\mathcal{F}$: **Nat** $\to$ **Art**. Top: five-scale biological hierarchy with assembly morphisms α_i and scale-specific reductions β_i. Bottom: engineered counterpart in **Art**, realised in additively manufactured materials.

This provides a five-scale hierarchy in **Nat** grounded in established wood cell-wall mechanics, which $\mathcal{F}$ maps to an additively manufactured bilayer by assigning the constitutive asymmetry at the fiber scale to raster orientation in the deposited layer.

The framework is generative. Given fiber-scale objects $\{F_i\}_{i\in I} \subset$ **Nat** indexed by stimulus type and tissue-level reductions $\{\beta_j\}_{j\in J}$ indexed by kinematic observable, each pair (i, j) for which the diagram

$$F_i \xrightarrow{\alpha_1} L \xrightarrow{\alpha_2} T \xrightarrow{(\beta_j,\,\mathrm{id})} S_{ij} \xrightarrow{\alpha_3} O_{ij}$$

is well-defined in **Nat** yields a valid engineered target $\mathcal{F}(O_{ij}) \in$ **Art** by functoriality, from which verified fabrication specifications in $\pi^{-1}(\mathcal{F}(O_{ij}))$ can be constructed. Combining verified components from distinct instantiations — different stimulus physics and different kinematic reductions [3] — produces new behavioral targets solely through composition, without any additional derivation. This includes combinations with no direct biological analogue, which arise as a structural consequence of functoriality rather than by design.

Computational implementation. The pipeline is implemented as a directed graph of Python components in Grasshopper (Rhinoceros 3D), with each node corresponding to one categorical layer and each edge carrying a parameter flow between functors. Swapping a fiber-scale object or tissue-level reduction propagates automatically to a new verified G-code output. All four designs are fabricated by fused filament manufacturing and experimentally validated, demonstrating that compositionally generated behavioral targets are physically realisable.

References

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