THE ROSEN FIBRATION

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In a long series of papers dating back to 1958 (Rosen, 1958a,b), and later condensed in the 1990s (Rosen, 1991), Robert Rosen introduced the (Metabolism, Repair)-systems paradigm. (M, R)-systems are generally recognized as the first application of category theory to a scientific field outside of pure mathematics; specifically, they were trying to represent the minimal functional relationships required for life. According to Rosen (1971), by making A, B, and f sufficiently complex, it is possible to reduce every (M, R)-system to the following diagram:

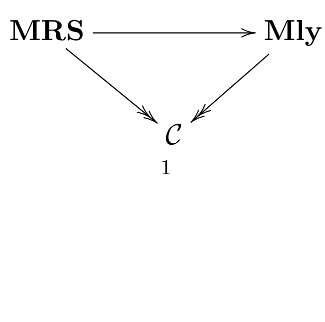
$$A \xrightarrow{f} B \xrightarrow{\Phi_f} [A, B], \tag{1}$$

where $[A, B]$ is the internal hom of a monoidal closed structure on a category $\mathcal{C}$. Historically, Rosen's early papers predate the widespread use of category theory (Varenne, 2013). Today, after thirty four years of Rosen (1991)'s synthesis, it is time to re-examine his vision through the lens of modern category theory, reformulating his ideas using fibrations, coalgebras for endfunctors, toposes of monoid actions, automata theory, and techniques from representation theory. By leveraging these structures, we can clarify Rosen's original insights and reveal new, elegant patterns within his conceptual framework. Our contribution particularly focuses on redeveloping diagrams 'of MR type' such as (1), treating them as elements of a *fibre* over (A, B), and observing that the assignment $(A, B) \mapsto \mathrm{MRS}(A, B)$ can be promoted to a profunctor $\mathrm{MRS} : \mathcal{C}^{\mathrm{op}} \times \mathcal{C} \to \mathbf{Set}$. From this profunctor one constructs a total category

$$\mathbf{MRS} \longrightarrow \mathcal{C} \times \mathcal{C},$$

(the *tabulator* of the profunctor) which we call the *Rosen fibration* of the title. In modern terms, the theory of (M, R)-systems is then largely the study of category-theoretic properties that this fibred category has and lacks. Such a new conception of Rosen's approach let us connect multiple findings in the literature. For instance, Băianu and Marinescu (1974) were the first to notice that Φ_f in an (M, R)-system is not a single map, but a component of a natural transformation $\Phi : (-) \Rightarrow [A, -]$. For us, then, an (M, R)-system *based at* A is taken to be a pair (f, Φ) where $f : A \to B$ and $\Phi : (-) \Rightarrow [A, -]$ is natural. For fixed input A the fibre of A-based (M, R)-systems is characterized universally as a certain category of parametric algebras, guaranteeing that its limits are created in the base; under standard hypotheses (e.g. local presentability) the fibres thus inherit completeness and accessibility properties.

Similarly, following Rosen (1964a,b)'s inquiries on how to connect automata theory with (M, R)-systems, Arbib (1966) showed that MR systems can be represented as special instances of Mealy automata, with state space $[A, B]$; this can be promoted to a functor ρ fitting in a commutative triangle



where **Mly** is a total category in which Mealy automata of all input and output types live, following Loregian (2025). Moreover, ρ is an epi-fibration.[1] Both properties are concise rephrasings of findings by Rosen (1964a,b); Arbib (1966); Warner (1982), and could be useful to develop new bio-inspired models of computation (Mossio et al., 2009; Palmer et al., 2016) and formally ground self-referential AI architectures (Zhang et al., 2025).

In addition, note that Rosen formulated his theory under the large assumption that all categories can be presented as subcategories of **Set** (MacLane and Eilenberg, 1945, p. 292-294). As a direct consequence, diagrams of MR type have a distinct set-theoretic / Cartesian flavor:[2] the objects $[A, B]$ are thought of as internal hom of a Cartesian-closed structure. Due to how biological processes occur in a world where resources are finite, this is an extremely strong, illegitimate assumption (Hofmeyr, 2021). *Affine* monoidal settings (Fritz, 2020), where duplication is not always possible[3], are much more apt to the description of biological phenomena. In order to represent the characteristic self-producing nature of life, Rosen proposed the existence of $a_0 \in A$ with $\Phi_f(f(a_0)) = f$. Such a condition imposes strict mathematical constraints on the category $\mathcal{C}$, and on its internal language. A central question is to identify the minimal conditions on a category $\mathcal{C}$ under which (M, R)-systems can be constructed (López-Díaz and Gershenson, 2025), and to characterize their relationship to semantic closure (Pattee, 1995) and the calculus of self-reference (Varela, 1975).

The naturality assumption on Φ_f made above allows one to apply a standard Yoneda-type argument, reducing organizational closure to a condition on the only component $\Phi_{1_A} : A \to [A, A]$. Thus, the biological capacity for self-repairing is encoded at the level of the input object A rather than being an accidental property of a single metabolic map. Following the early insights provided by Băianu and Marinescu (1974), we close our contribution by showing how one can extend all the above observations (the presence of a fibration, co/completeness, coalgebraic presentations, organizational closure conditions and obstructions thereof...) to study MR chains of n steps,

$$A \to B \to [A, B] \to [B, [A, B]] \to [[A, B], [B, [A, B]]] \to \cdots ,$$

following a Fibonacci-type recurrence, the $n + 2$ object M_{n+2} in the chain equals the internal hom $[M_n, M_{n+1}]$. If time allows, we will also outline how this leads to an approach via the well-established representation theory of quivers.

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[1] This means that one finds Cartesian lifts only for epimorphisms in its codomain.

[2] here in the sense of Cartesian-monoidal.

[3] In logical terms, where the contraction rule is dropped but the weakening rule remains; in categorical terms, categories where there isn't always a diagonal map $X \to X \otimes X$ but projections continue to exist.

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